KINE 601

Regression & Non-parametric Statistics

Reading: Huck pp 416 - 513

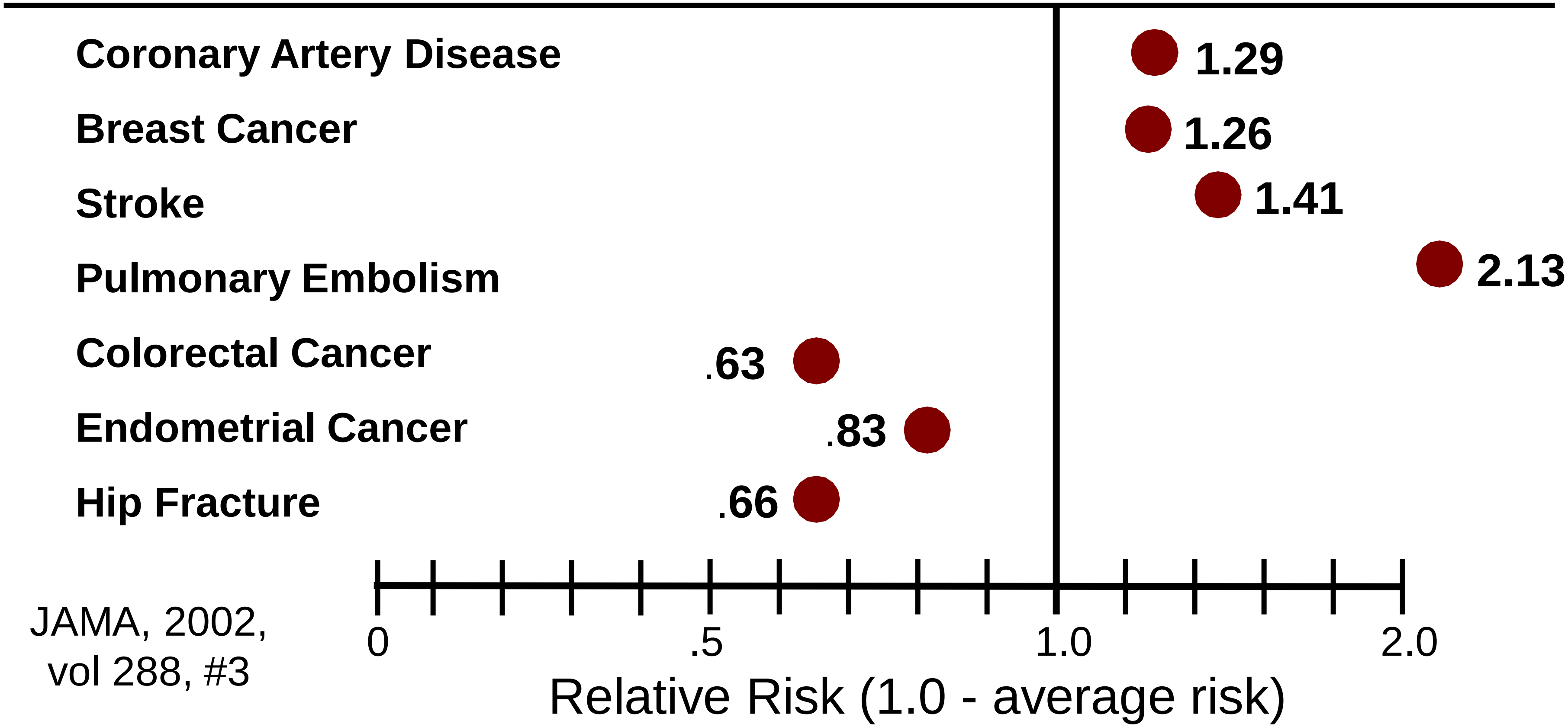
Regression - the statistic of prediction

- **Types of Regression:**
  - bivariate (two variable) regression - also called **simple linear regression**
    - similar to Pearson correlation
    - one **independent** or **predictor** variable, one **dependent** or **criterion** variable
  - multiple regression
    - more than one independent variable, one dependent variable
  - logistic regression
    - using one or more independent variables to predict dichotomous classification
    - example: using blood pressure, cholesterol levels, and whether or not one is a diabetic to predict whether or not a person will have a heart attack within the next year **(similarly, the odds ratio or relative risk of having a heart attack in the next year can be predicted)**
- **Uses of regression:**
  - prediction
    - examples: predicting GRE scores from undergraduate GPA
      predicting changes in HDL-C with exercise from fitness & lipid variables
  - explanation (which is, in fact, “backward prediction”)
    - example: using attitudes toward health to explain why people exercise
      using family & health belief information to explain why people smoke

Recent Use of Logistic Regression

CVD Risk, Breast Cancer, and Hormone Replacement The New and Current Controversy

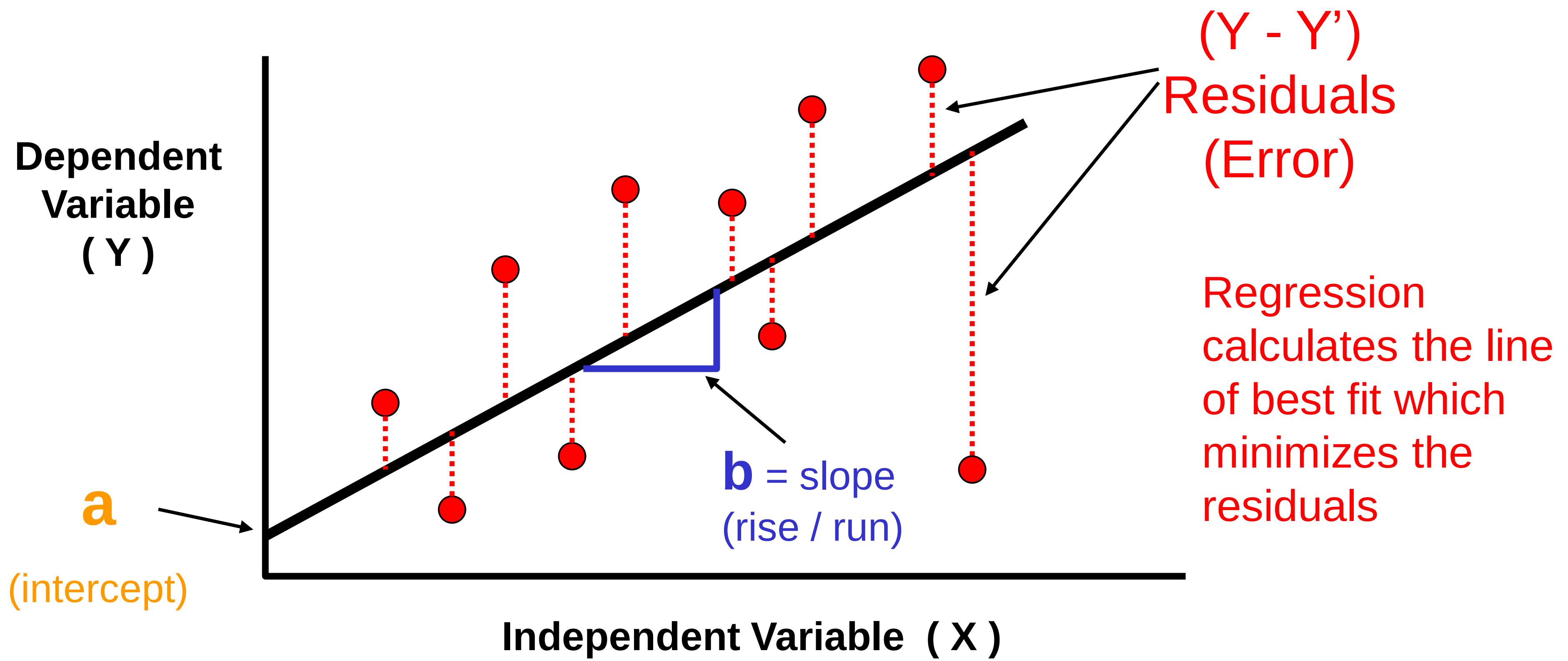
Results from the Women's Health Initiative - E2 + Progesterone



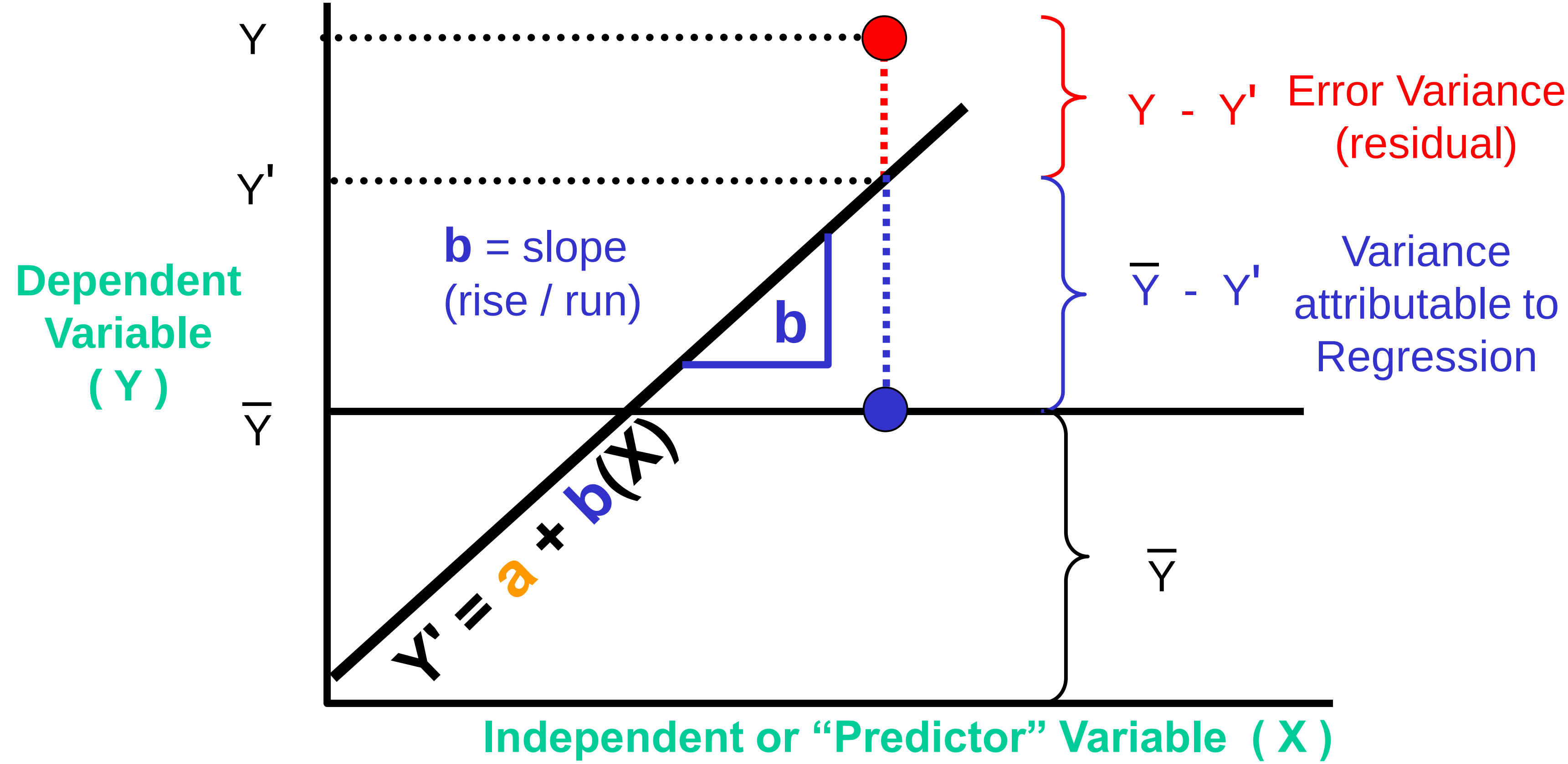
JAMA, 2002, vol 288, #3

Simple Linear Regression

- **the regression line: Y' = a + b (X)**
  - **Y'** is the predicted value of the dependent variable
  - **a** is a constant - represents where regression line intercepts the vertical axis
  - **b** is non-standardized regression coefficient - slope of regression line
  - **X** is the known score or number of the independent or "predictor" variable



Relationship between ANOVA and Regression



If we made the analysis above for each data point, we could arrive at a sum of squares then a variance (mean square) for both error and the regression. This would facilitate an **F - ratio** that could be analyzed to test the hypothesis that **b** is significantly different from 0.

$$F = \frac{\text{regression MS}}{\text{error MS}}$$

Example of Simple Linear Regression

- A researcher wishes to determine if systolic blood pressure can be predicted using weight. Twenty subjects were recruited and were assessed for weight and systolic blood pressure. Simple linear regression analysis was performed to test the null hypothesis that no relationship exists (**b** = 0).

Variable: SBP

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	4210.38190	4210.38190	54.538	0.0001
Error	18	1389.61810	77.20101		
C Total	19	5600.00000			

Root MSE	8.78641	R-square	0.7519
Dep Mean	140.00000	Adj R-sq	0.7381
C.V.	6.27601		

Parameter Estimates

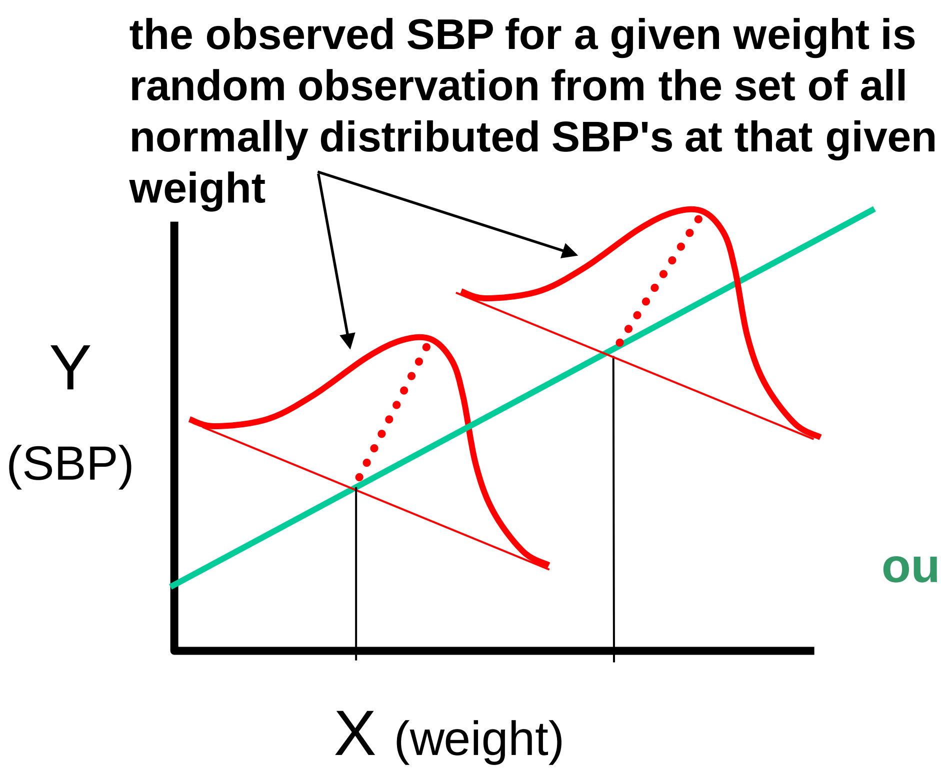
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	75.754597	8.91856647	8.494	0.0001
WEIGHT	1	0.378359	0.05123363	7.385	0.0001

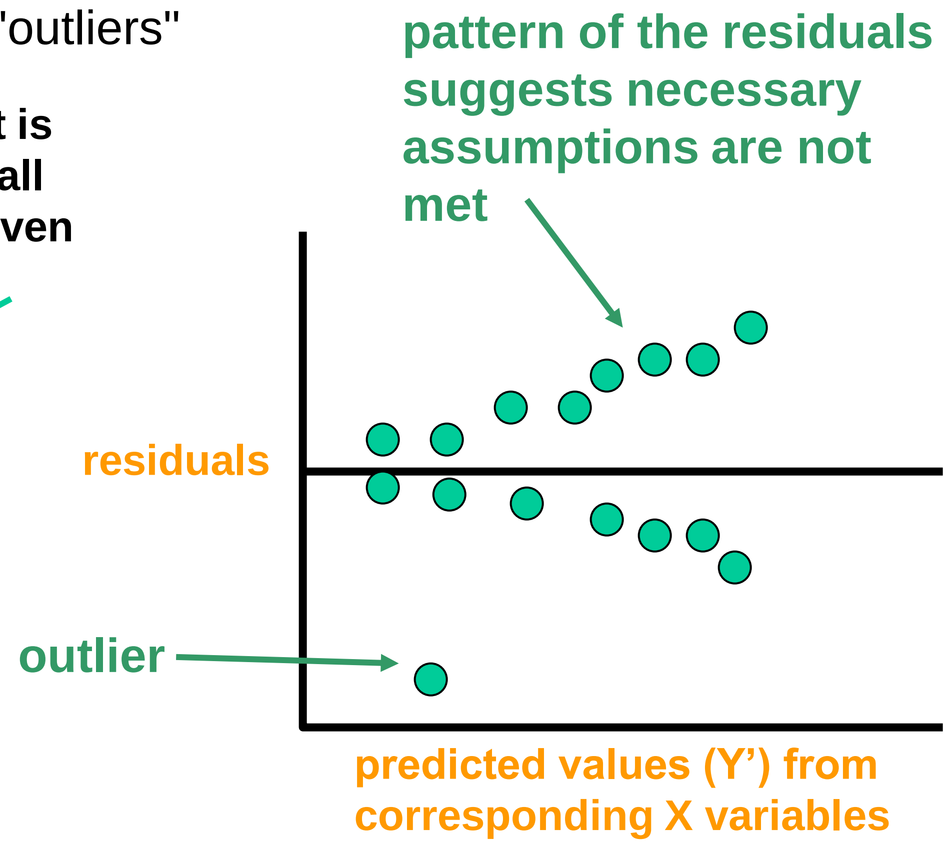
Standard error of the estimate - the standard deviation of the residuals.

The “smaller” the SEE, the smaller the variability in the residuals and the better "fit" the regression line

Simple Linear Regression

- **Assumptions for regression:**
  - for each value of the predictor variable, the possible true values for the corresponding observed dependent or “criterion” variable are normally distributed and have homogeneous variances.
    - can be checked by plotting predicted values and residuals
      - also a good way to spot "outliers"





Multiple Regression

- <u>**the regression line:**</u> **$Y' = a + b_1(X_1) + b_2(X_2) + b_3(X_3)\ldots$**
  - **Y'** is the predicted value of the dependent variable
  - **a** is a constant - represents where regression line intercepts the vertical axis
  - **b** is regression coefficient - represents the <u>mathematical</u> contribution of the corresponding X variable to the overall regression
    - the sign of the **b** coefficients describes the nature of the relationships (direct or inverse) of the corresponding predictor variable with the criterion variable
    - **caution:** the **b** coefficients must be <u>standardized</u> before comparisons can be made regarding the amount of variation contributed by each corresponding predictor variable. This will not be accurate however, if there is a significant relationship between any of the predictor variables
      - <u>**collinearity**</u> or <u>**multicollinearity**</u>
    - standardized regression coefficients are called beta weights and are represented by the greek letter **β**
  - **X's** are the actual known score - the predictor variable values
    - <u>**dummy variables**</u>: coding 0's or 1's for dichotomous variables (gender)
      - dummy variables other than 0's or 1's are often used but should not be because the numbers (1,2,3,4,…etc.) have no quantitative meaning; ie. you cannot assume that the dependent variable changes twice as much with a dummy coded predictor variable with a **X** of 4 as with a dummy coded **X** of 2.

Example of Multiple Linear Regression

- A researcher wishes to determine if systolic blood pressure can be predicted using weight and total cholesterol. Twenty subjects were recruited and were assessed for systolic blood pressure, weight, and total cholesterol. Multiple linear regression analysis was performed to test the null hypothesis that weight and total cholesterol have no predictive value (all **b**'s = 0).

Variable SBP

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	4320.70287	2160.35143	28.708	0.0001
Error	17	1279.29713	75.25277		
C Total	9	5600.00000			

Root MSE	8.67484	R-square	0.7716
Dep Mean	140.00000	Adj R-sq	0.7447
C.V.	6.19631		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	122.761115	39.80913557	3.084	0.0067
WEIGHT	1	0.230775	0.13197030	1.749	0.0984
TCHOL	1	-0.090914	0.07508677	-1.211	0.2425

Types of Multiple Regression

- **Nonlinear Regression: $Y' = a + b_1 (X_1) + b_2(X_2)^2$**
  - also referred to as **polynomial regression**
  - used if a non-linear (curvalinear) relationship is suspected between predictor variables & the criterion variable
    - example: the relationship between age and maximal bench press poundage would be a parabolic (inverted "**U**") relationship with an equation similar to the one above.

- **Stepwise Multiple Regression**
  - variables are entered into the regression one at a time in an effort to determine which variable contributes most to the regression ("gives the most bang for the buck")
    - usually, the coefficient of determination (R^2) for each equation is compared to determine if adding additional variables to the model significantly increases the capability of the model to predict the criterion variable. Other statistics such as Mallows CP may be used to determine which regression model is best.

Non-Parametric Statistics

- **Non Parametric Statistics:** statistics that do not require the dependent variable to be normally distributed (there may not even be a true dependent variable)
  - examples of variables that are not normally distributed:
    - eye color, % of people who own a lawn mower, the academic rank of class members, political and moral opinions, the number of meals a person eats every year containing < 30% fat

- **Advantages of Non-Parametric Statistics:**
  - no restriction of normality and variance homogeneity
  - computations (even by hand) are quick and speedy

- **Disadvantages of Non-Parametric Statistics:**
  - less "powerful" than parametric statistics
    - more subjects needed to reject the null hypothesis
      - remember - statistics with the most power are parametric statistics when assumptions are met
  - since only nominal or ordinal data can be used, these methods may not fully utilize all of the information contained in the data

Non-Parametric Statistics

- **The Chi Square test - type 1: goodness of fit**
  - tests whether or not the frequencies or proportions found in the categories of some nominal (categorical) variable fit some pre-conceived or expected pattern
    - are the frequencies equal? do they follow hypothesized proportions?
  - example: a researcher wishes to determine if the percentage of people with high blood pressure (SBP > 140) is equally distributed among a sample of Caucasians and African-Americans (normally 1 in 5 adults are hypertensive)

```
RACE     BPSTATUS

Observed ,
Expected ,(based on 1/5 x 9 =1.8)
    ,
    ,HYPER  ,NORMO  , Total
ffffffffff^ffffffff^ffffffff^
AA    ,    9 ,    0 ,    9
    ,  1.8 ,  7.2 ,
    ,     ,     ,
    ,     ,     ,
ffffffffff^ffffffff^ffffffff^
C     ,    2 ,    7 ,    9
    ,  1.8 ,  7.2 ,
    ,     ,     ,
    ,     ,     ,
ffffffffff^ffffffff^ffffffff^
Total      11     7     18
```

$$\chi^2 = \Sigma \frac{(\text{observed - expected})^2}{\text{expected}}$$

$$\frac{(9 - 1.8)^2}{1.8} + \frac{(0 - 7.2)^2}{7.2} + \frac{(2 - 1.8)^2}{1.8} + \frac{(7 - 7.2)^2}{7.2}$$

$$= 36.03$$

```
Statistic   DF Value  Table Value (.05)
ffffffffffffffffffffffffffffffffff
Chi-Square  1  36.03      3.84
```

Since **the calculated value** exceeds the **table value** reject the null hypothesis that hypertension is equally distributed among Caucasians and African Americans in this sample

Non-Parametric Statistics

- **The Chi Square test - type 2: test of independence (contingence)**
  - tests whether or not two categorical variables are independent of one another
    - example: a researcher wishes to determine if race and family history are independent of one another with regard to heart disease risk
      - H_0: race (RACE) is independent of family history (FAMHIST)

FAMHIST

```
Observed ,  →
Expected , (row total x column total) / grand total
Percent  ,
Row Pct  ,
Col Pct  ,1      ,2      ,3      , Total
ƒƒƒƒƒƒƒƒƒˆƒƒƒƒƒƒƒƒˆƒƒƒƒƒƒƒƒˆƒƒƒƒƒƒƒƒˆ
AA       ,     2 ,     2 ,     5 ,     9
         ,     4 ,   2.5 ,   2.5 ,
         , 11.11 , 11.11 , 27.78 ,  50.00
RACE     , 22.22 , 22.22 , 55.56 ,
         , 25.00 , 40.00 ,100.00 ,
ƒƒƒƒƒƒƒƒƒˆƒƒƒƒƒƒƒƒˆƒƒƒƒƒƒƒƒˆƒƒƒƒƒƒƒƒˆ
C        ,     6 ,     3 ,     0 ,     9
         ,     4 ,   2.5 ,   2.5 ,
         , 33.33 , 16.67 ,  0.00 ,  50.00
         , 66.67 , 33.33 ,  0.00 ,
         , 75.00 , 60.00 ,  0.00 ,
ƒƒƒƒƒƒƒƒƒˆƒƒƒƒƒƒƒƒˆƒƒƒƒƒƒƒƒˆƒƒƒƒƒƒƒƒˆ
Total          8       5       5      18
           44.44   27.78   27.78  100.00
```

Contingency Table

```
            DF  Value  Prob
ƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒƒ
Chi-Square  2  7.200  0.027
Phi Coeff.      .632
```

Since ***p*** < .05 we reject the null hypothesis that race is independent of family history

The special phi coefficient shows the strength of association between **RACE** & **FAMHIST** to be moderate

Non-Parametric Statistics

tests of significance for ordinal data

- **Mann Whitney *U* - test**
  - test for two independent samples of data that are in rank (ordinal) form
    - tests the <u>medians</u> of samples for significant differences
    - results in a z-score which can be compared to table values for a given level of α
    - non-parametric analog of an independent ***t*** - test
    - can be used for continuous data that are known not to be normally distributed
- **Wilcoxin sign rank test**
  - test for two dependent samples of data that are in rank (ordinal) form
  - non-parametric analog of an dependent or correlated ***t*** - test
- **Kruskall-Wallace test**
  - test for two or more independent samples that are in rank (ordinal form)
  - results in an ***H*** statistic that can be tested with a χ^2 distribution
  - non-parametric analog of the one way ANOVA
- **Friedman test**
  - test for two or more samples in rank (ordinal) form that are correlated
  - results in a χ^2 statistic that can be tested with a χ^2 distribution
  - non-parametric analog of a one-way repeated measures ANOVA